40002 Maths Methods

Pain

Anyone fancy trying to format matrices in Word for Linear Algebra lol

2ai) Supremum = (not in set)

Infimum = (not in set)

2aii)

Given we want to find the limit and some such that , ,

(1) (should be: |an – l| but word doesn’t like absolute values it seems)

We speculate that the limit is 0, so set

Now,

<-> (2) (should have | | around LHSs)

<-> (3) (step: abs magnitude removed)

<-> (4) (don’t need to expand gbrackets)

<-> (5) (finally some good formatting)

<-> (6)

<-> (7) (bada bing bada boom)

So, if we choose

Then,

We have found an and have shown

(from (2) - (7))

This satisfies (1) and proves the limit of is equal to 0 (and hence is convergent) by the definition of limits. (put that funky lil black square here and you’re done)

2aiii)

Notice that:

And

Hence L’Hôpital’s rule is applicable here:

(Because )

2bi) Left as an exercise to the reader in using ~~Wolfram Alpha 😉~~

(This is a telescoping series with partial sum sqrt(k+1), so it’s not bounded above)

In order to apply the integral test, 3 conditions must be met:

* Continuous for : yes
* Positive for : yes
* Decreasing for : Let

: yes

We can now apply the integral test:

… (will fill the steps in if I find the will to)

By definition of the integral test, if the result of the integral is the series converges, otherwise it converges to . Hence, the sum converges to .

***Alternate Solution***

Therefore, applying the series notation on both the LHS and RHS, we know that if the bounded (RHS) series diverges, then so does the LHS series (original series). Therefore, from standard divergent series (given in the lecture notes idk), the RHS series diverges. Therefore, the original series (LHS) must diverge too.

**Alternate Solution**

Proof that converges to :  
For any :

By taking we have that for all , . So converges to .

So converges to .

2bii)

We apply the nth root test to determine the limit of the series:

(the bracketing is awful, should be: | (frac)n | 1/n )

(should have abs magnitude around it)

Therefore, by definition of the nth root test, the series converges.

2biii)

Since c = 1.1 > 1, converges for all c > 1, therefore converges.

2c(i)

Since the formula for sum of geometric series is ,

From this, we can calculate the radius of convergence using nth root test. Seems like we could cut out all the work below.

…

Hence,

(How did you all calculate the radius of convergence for this series btw)

(Did anyone try the nth-root test on this)

So,

Therefore radius of convergence is

(if need more explanation thingie)

, so same as

same as

same as

2cii)

Applying chain rule we get

G(0) = f(0)

G'’(0) = pi^2 \* f’(0)

G'’’’(0) = p^4 \* (3f’’(0) - f’(0)) (is there an easier way to get to this step?) *(just a lot of derivatives sadly, I don’t think there’s an easier way) (there is an easier way #ratio)*

So g(x) = f(0) + 1/(2!)\*pi^2\*f’(0)\*x^2 +1/(4!)\* p^4 \*(3f’’(0) - f’(0))\*x^4 +…

F(x) radius of convergence when abs(x) < ½

So g(x) = f(1-cos(pi \* x)) radius of convergence when 1 – cos(pi\*x) < ½

Which is when abs(x) < 1/3 *(questioning whether this also converges for 5/3 < x <= 2?) [I guess no, because f(x) only converges with radius ½ around x = 0 ? ] (I'm not sure that matters, I'll show my working below \*) (oh wait nvm, you’re right)*

(\* Consider that f(x) having a radius of convergence of ½ implies that , and thus we have that the radius of convergence of g(x) can be deduced as follows: and from this we note that g(x) thus converges when which should give a pretty wack radius of convergence)

(Also converges adding integer multiples of 2\*pi but the radius of convergence is still 1/3)

LINEAR ALGEBRA PART 1

1ai)

R2 = R2 + R1

R3 = R3 – 2R1

~

R3 = R3 – 5R2

Then R3 = 1/16 R3

~

R1 = R1 – R2 - R3

R2 = R2- 3R3

~

Is row echelon form

1aii)

x1 – x4 = 1. -> x1 = 1 + x4

x2 – x4 = 0. -> x2 = x4

x3 – 2x4 = 1. -> x3 = 1+ 2x4

-> x4 =. x4

x4 is the free variable

Basis is .

# the coefficient of x4

1aiii)

A line that passes through point [1 0 1 0] and is parallel to [1 1 2 1]

1aiv) Distance from to :

has a minimum at , so has a minimum at .

This gives:

A more linear algebraic attempt:

Shortest distance will be the perpendicular distance from origin to the line.

Hence, we try to find the point on the perpendicular intersection.

Let point be [1+λ, λ, 1+2λ, λ]. This means that the dot product with [1,1,2,1] = 0

(1 + λ) + λ + 2(1+2λ) + λ = 0

7λ = -3

λ = -3/7

1bi)

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | -1 | 2 | 1 |
| 2 | 1 | -1 | 3 |
| 1 | 5 | -8 | 3 |

~

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0 | 1/3 | 4/3 |
| 0 | 1 | -5/3 | 1/3 |
| 0 | 0 | 0 | 0 |

Rank (A) = 2 # number of independent columns

Nullity(A) = 4 – 2 = 2. # order – rank

(I think the nullity is 2??)

1bii)

Image space = columns where there are leading 1’s

Im(A) = {[1 2 1], [-1 1 5]}

Null(A) = {[-1/3 5/3 1 0], [4/3 1/3 0 1]} # solution basis

1biii)

ci)

For :

For each of , the eigenvectors are found by:

Giving:

cii)

For any , :

For , ,

For , ,

For , ,

ciii)

(Or any other ordering of the eigenvectors)

d)